

University of Groningen

## The arithmetic of maximal curves, the Hesse pencil and Mestre curve

Anema, Ane Schelte Izaäk

**IMPORTANT NOTE:** You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2016

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Anema, A. S. I. (2016). *The arithmetic of maximal curves, the Hesse pencil and Mestre curve*. [Thesis fully internal (DIV), University of Groningen]. Rijksuniversiteit Groningen.

### Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

### Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

## Propositions accompanying the PhD thesis

### **The arithmetic of maximal curves, the Hesse pencil and the Mestre curve**

by *Ane Anema*

1. Let  $C$  be a curve of genus  $g$  over  $\mathbb{F}_q$  with non-supersingular Jacobian variety. The upper bounds presented in Chapter 1 of the thesis for the case  $g = 1$  on the degree of  $k/\mathbb{F}_q$  such that  $C$  is maximal over  $k$  generalize to the case  $g > 1$ .
2. For infinitely many primes  $p$  there exists a genus 2 curve  $C$  over  $\mathbb{F}_p$  such that  $C$  is maximal over  $\mathbb{F}_{p^3}$ .
3. The hyperelliptic curve  $C$  over  $\mathbb{F}_{17}$  defined as  $y^2 = x^9 + 7x^5 - x$  is maximal over  $\mathbb{F}_{17^3}$  and the twist of  $C$  defined as  $y^2 = x^9 + 11x^5 + x$  is minimal over  $\mathbb{F}_{17^3}$ .
4. The work of Katz on Frobenius trace ratios in the Legendre family of elliptic curves over finite fields does not carry over in a natural way to the Hesse pencil.
5. There are at most countably many  $j \in \mathbb{C}$  such that the Jacobian variety of the curve  $D_{a,b}$  is isogeneous to a product of elliptic curves, where  $a, b \in \mathbb{C}$  satisfy  $j = 1728 \frac{4a^3}{4a^3 + 27b^2}$  and  $D_{a,b}$  is defined as

$$y^2 = (x^3 + ax + b)(ax + b)(ax - 3b).$$

The following two propositions are related to Chapter 6 of the thesis:

6. The maximal exponent 4 extension of  $\mathbb{Q}$  unramified outside 2, 5 and  $\infty$  has degree  $2^{14}$  over  $\mathbb{Q}$  and is the splitting field of
$$(x^8 + 4x^6 + 4x^4 - 2) \cdot (x^8 + 4x^6 + 4x^4 - 5) \cdot (x^8 + 4x^6 + 4x^4 - 10) \\ \cdot (x^{16} + 6x^{12} - 4x^{10} + 8x^8 + 8x^6 - 4x^4 - 8x^2 + 4).$$
7. The number of isogeny classes of abelian surfaces  $A$  over  $\mathbb{Q}$  with good reduction at every prime  $p \neq 2, 5$  and  $\mathbb{Q}(A[2])/\mathbb{Q}$  a 2-extension is at most  $2.1 \cdot 10^{1435}$ .
8. The scientific community would benefit from a professionally maintained collection of errata of scientific publications.